Uniform Standard for Teaching Foundational Principles in Statics and Dynamics, Momentum Perspective

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I. Introduction

It has become clear in the years that we have been teaching engineering subjects that there is a discontinuity between how related subjects are taught. By that, we mean that fundamental principles of mass, energy and momentum are indifferent to the application, yet they are introduced and utilized very differently in various engineering courses. One good example of this disparity is the momentum principle, which is a combination of Newton's 2nd Law of Motion and Reynolds Transport Theorem. In every statics and dynamics textbook, there is no mention of the momentum principle, even though, as will be shown in this paper, many problems (and entire chapters) can be best understood utilizing the momentum principle.

Most introductory statics and dynamics courses do teach various forms of "momentum methods." The plural form of "momentum methods" indicates that there is more than one, which can be observed in most statics and dynamics textbooks. Unfortunately, these methods are only special-cases of the general momentum principle. In fact, at no time in any statics and dynamics textbook that we have seen, is the full momentum principle utilized.

The problem with not utilizing the full momentum principle is two-fold. First, there are many "momentum methods" introduced in textbooks unnecessarily overcomplicating a simple principle. Secondly, as a student progresses from one course to another, they virtually have to relearn what the momentum principle really is, almost as if it's something brand new.

It is the intent of this paper to show, through example, that the same basic form of the momentum principle should be utilized throughout the curriculum, starting with basic statics and dynamics and progressing into the fluid and thermal sciences (fluid mechanics) and the rest of the curriculum.

II. Overview of Course

The *Engineering Mechanics* course at Oakland University mainly consists of sophomore-level undergraduate students. The course is a four-credit class, and involves both lecture and laboratory, hands-on components. The lectures introduce new fundamental principles in the curriculum which most students will see as familiar (Newton's 2^{nd} Law of Motion, energy, etc.). The lectures serve to introduce and apply these principles to problems in statics and dynamics. The laboratory component is strictly geared toward exploration, utilizing knowledge, and comprehension learned in this very course.

The *Engineering Mechanics* course, previously called *Dynamics and Vibration*, was instituted in the 1970's to be the primary fundamental classical mechanics experience for introductory engineering students. As a four-credit course, the class meets twice a week for approximately an hour and a half. The lectures consist of a variety of introductions and plenty of example problems, plus collaborative student exercises. The lecture is thus broken up to include regular breakout sessions involving active learning techniques, student-centered learning and collaborative learning. Homework is assigned regularly to keep skills sharp and up-to-date.

The laboratory component is designed to nurture visualization and stimulate creativity. Rather than rely purely on simulation, the laboratory experience was made to be a hands-on experience. In fact, as part of the laboratory experience, the student groups actually assemble some of the systems by hand before running an experiment.

III. Course Goals and Objectives

Oakland University catalogue course description:

> *"Statics and dynamics of particles and rigid bodies: analysis of trusses, frames, beams, centroids and moments of inertia; kinematics, Newton's Second Law, work and energy, linear and angular impulse and momentum. With laboratory."*

The course objectives, as determined by the departmental undergraduate committee, are as follows:

1. Be able to explain and apply the kinematics of particles in Cartesian, curvilinear and intrinsic coordinate systems.
2. Be able to apply Newton's Second Law to particle motion in Cartesian, curvilinear and intrinsic coordinates.
3. Be able to define and apply to particle motion: work of a force, principle of work and energy, potential energy, principle of impulse and momentum, impulsive motion, and motion under direct and oblique impacts.
4. Be able to describe the motion of systems of particles: kinematics, kinetics including linear and angular momentum, and energy methods.
5. Be able to define and apply the kinematics of plane motion of rigid bodies, including angular velocity, rotation about fixed axes and general plane motion.
6. Be able to apply Newton's Second Law to the plane motion of rigid bodies.
7. Be able to apply work/energy and impulse/momentum methods, including problems involving impact, to plane motion of rigid bodies.

The major goal of this statics and dynamics course is to expose students to, and challenge them to think about, the entire taxonomy of the analysis process for a variety of systems either moving or stationary, open or closed. The lectures, including student-centered and active learning techniques, promote knowledge, comprehension and application. Regular homework and frequent small quizzes further promote these important aspects of the learning process. The hands-on laboratory experiences then take students through the analysis, data interpretation and evaluation.

IV. Momentum principle versus "momentum methods"

Although Newton's 2nd Law of Motion is utilized throughout every statics and dynamics textbook, it has a limitation – it is formulated for a closed particle, not for more general systems that are neither particles nor closed. For open systems where, in general, the mass of the system may change (as well as the velocity of the center of mass), due to mass transferred across system boundaries, Newton's 2nd Law is combined with Reynold's Transport Theorem. The result is usually introduced in introductory fluid mechanics courses in integral form as follows:

$$\frac{d}{dt}\int_V \rho\vec{v}\,dV = \int_V \rho\vec{f}_b dV + \int_A \vec{f}_s\,dA - \int_A \rho\vec{v}(\vec{v}\cdot\vec{n})\,dA \qquad (1)$$

In equation 1, v is the velocity, ρ the density, f_b the body force per unit mass, f_s the surface force per unit area. Equation 1 is generally referred to as either the conservation of momentum or the momentum principle. These authors prefer the latter as momentum is generally not conserved (think mashed potatoes thrown against a wall), only energy is.

What the authors of the current paper have not seen in any available textbook, is the lumped parameter formulation of this momentum principle, which should be stated in equation form as follows:

$$\frac{d\vec{M}}{dt} = \vec{F}_b + \vec{F}_s + \vec{F}_m \qquad (2)$$

There are three distinctly different external forces that may change the momentum of any system: surface forces (pressure, shear stress, etc.), body forces (gravity, electric and magnetic forces, buoyancy, etc.) and forces due to momentum transfer with mass transfer (thrust from a fluid jet, jet or rocket engines, etc.).

For closed systems, the last term is zero by virtue of no mass (and therefore no momentum) crossing the boundary of the system. In this case, the time rate of change of system momentum – the left-hand term which consists of the product of mass and velocity – becomes the familiar product of mass and acceleration as mass can be taken out of the derivative (it is constant by virtue of the conservation of mass principle). Thus, equation (1) reduces to:

$$M\vec{a} = \sum \vec{F}_b + \vec{F}_s = \sum \vec{F} \quad (3)$$

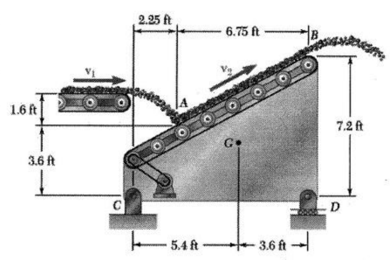
This is the equation that is utilized in most if not all statics and dynamics textbooks when beginning dynamics problems whether open or closed, and essentially reverse engineering by reasoning some elements of (2). On the other hand, almost without exception fluid mechanics textbooks begin with equation (1) which is the integral for of (2). In both topical areas, it is these authors opinion that equation (2) ought to be used at least at first. Doing so would bridge the areas of solid mechanics with the fluid and thermal sciences, which would be an advantage in teaching statics and dynamics since it is typically an earlier course in the curriculum. Conversely, utilizing the lumped parameter formulation of the momentum principle would give students in fluid mechanics a deeper understanding of it, just like students introduced into thermodynamics exclusively utilize the lumped parameter formulation of the conservation of mass and energy principles.

V. An open system example

There are many possible examples to utilize here proving out the idea that utilizing the lumped parameter formulation of the momentum principle is superior to the ‘momentum methods’ and such strewn throughout statics and dynamics books, but the most compelling case may be for open systems where mass is moving into and out of a system (either static as seen in many statics and dynamics textbooks, or moving as seen in many fluid mechanics textbooks).

An example of a typical open system problem is one where the supporting reaction forced need to be determined for a conveyor belt system with coal being dumped on, and then discharged, from said conveyor, or problem #14.69 in Beer et al [1] for which the solution below is found in [2].

The kinematics portion of the problem is similar to the way it is treated regardless of which approach is taken (although we make students derive the kinematic equations from scratch every time, but that is the subject of what should be another paper). The difference comes in when kinetics comes into play. From the Beer et al [2] solution manual:



PROBLEM 14.69

Coal is being discharged from a first conveyor belt at the rate of 240 lb/s. It is received at A by a second belt which discharges it again at B. Knowing that $v_1 = 9$ ft/s and $v_2 = 12.25$ ft/s and that the second belt assembly and the coal it supports have a total weight of 944 lb, determine the components of the reactions at C and D.

SOLUTION

Velocity before impact at A:

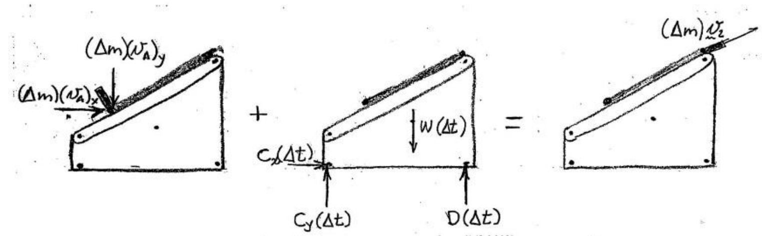
$$(v_A)_x = v_1 = 9 \text{ ft/s} \longrightarrow$$

$$(v_A)_y^2 = 2g(\Delta y) = (2)(32.2)(1.6) = 103.04 \text{ ft}^2/\text{s}^2 \qquad (v_A)_y = 10.151 \text{ ft/s} \downarrow$$

Slope of belt: $\tan\theta = \dfrac{7.2 - 3.6}{6.75}, \ \theta = 28.07°$

Velocity of coal leaving at B: $v_2 = 12.25(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$

Apply the impulse-momentum principle.



$\xrightarrow{+}$ x components: $(\Delta m)(v_A)_x + C_x(\Delta t) = (\Delta m)v_2\cos\theta$

$$C_x = \frac{\Delta m}{\Delta t}\left[v_2\cos\theta - (v_A)_x\right] = \frac{240}{32.2}(12.25\cos 28.07° - 9)$$

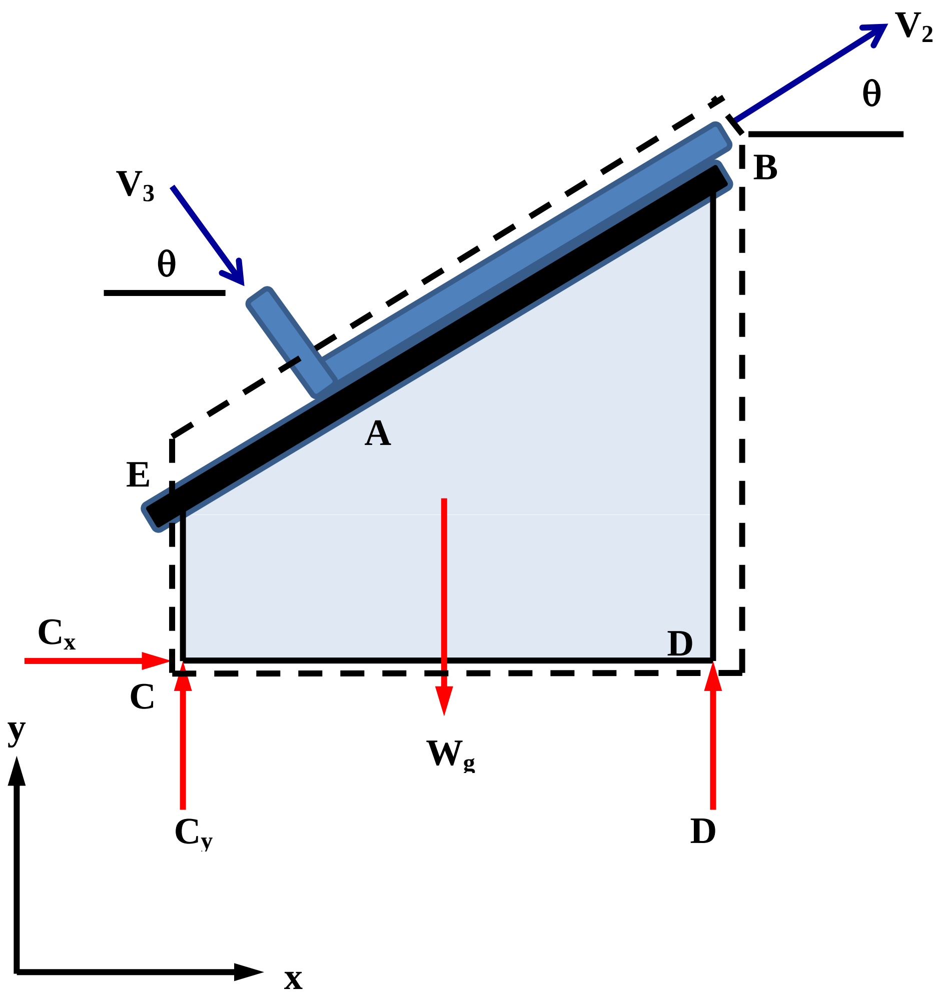
$$C_x = 13.48 \text{ lb}$$

$+\circlearrowleft$ moments about C:

$$(\Delta m)\left[-3.6(v_A)_x - 2.25(v_A)_y\right] + 9D(\Delta t) - 5.4W(\Delta t)$$

$$= (\Delta m)\left[-7.2\, v_2\cos\theta + 9v_2\sin\theta\right]$$

Alternately, a better way to perform the exact same problem as above would be to begin with eth full lumped-parameter formulation of the momentum principle, equation (2). Similarly to what is typically done with the vector form of Newton's 2nd Law of Motion, the momentum principle will be broken up into components and treated separately. The conveyor system as a whole is static and steady state (no property of the systems is changing with time); thus the left hand term which is the system's inertia, is zero no matter which component is being analyzed.



x-Direction.
In the x-direction, the body force (gravity) has no component in this direction and is thus zero. The only terms remaining then are the surface force, which is C_x, and the force due to momentum transfer, $\vec{F}_m$. Since the force due to momentum transfer is eth momentum flowrate, or the product of mass flowrate and the velocity (average velocity in the case of the lumped parameter formulation), the x-component of the momentum principle becomes:

$$C_x = \dot{m}_2 v_2 cos\theta - \dot{m}_3 v_{3,x} \quad (4)$$

And since the mass flowartes are exactly equal as a consequence of the conservation of mass,

$$C_x = \dot{m}(v_2 cos\theta - v_{3,x}) \quad (5)$$

y-Direction.
In the x-direction, the body force (gravity) has its entirety in it, but is negative in relation to the defined coordinate system. The surface force in the y-component of C in addition to the force D at that roller support. And then the y-components of the force due to momentum transfer have to be accounted for. Thus, with the mass flowrates being equal,

$$C_y + D = W_{G+}\dot{m}(v_{3,y} - v_2 \sin\theta) \quad (6)$$

Although there are enough parameter to determine Cx in equation (5), there are two unknowns in (6) and another equation is needed. There are no other non-trivial components of the momentum principle. Therefore, the angular form of the momentum principle must be used, and here again the lumped parameter formulation not seen in textbooks is as follows:

$$\vec{r}\ x \left\{ \frac{d\vec{M}}{dt} = \vec{F}_b + \vec{F}_s + \vec{F}_m \right\} \quad (7)$$

The system is static and in steady state so the transient term is zero Evaluation this about point C, it can be determined that:

$$\vec{r}_C\ x \left\{ \frac{d\vec{M}}{dt} = \vec{F}_b + \vec{F}_s + \vec{F}_m \right\}_C = \vec{0}$$
$$= \vec{r}_{CG} x(-W_G)\vec{j} + \vec{r}_{CD}\ xD\vec{j} + \vec{r}_{CD} x\dot{m}\left(v_{3,x}\vec{i} - v_{3,y}\vec{j}\right) + \vec{r}_{CA} x\dot{m}(v_2\cos\theta\vec{i} - v_2\sin\theta\vec{j}) \quad (8)$$

The force C is not a variable in (8) and thus it can be solved for the unknown reaction force D at the roller support, which in turn can be substituted back into (6) to solve for C_y. Once C_y is obtained, a vector expression can be obtained for the reaction force C.

It can be seen that the above yields the exact same answer as the one obtained by Beer et al. However, we used the full lumped version of the momentum principle in the same way as it is used in the fluid and thermal sciences, and although our solution is longer, that by itself is not necessarily a bad thing and indeed may add to a student's knowledge of this fundamental principle of nature. And that in the end is the whole point of the exercise – an understanding of basic principles throughout multiple courses.

It would be interesting to perform a sort of assessment of the advantage of the above pedagogical approach versus the traditional one employed in all statics and dynamics textbooks. However, it would be difficult to do one in any given class as students will undoubtedly be exposed to one method or the other only, and thus little basis for comparison. Performing some sort of assessment on the above pedagogical approach will be the subject of further research.

VI. Acknowledgements

The authors would like to acknowledge the many students who have participated in our many classes over the years. Their enthusiasm, creative thinking, and inquiring questions to us during their attempts to better understand the material, continually fuels our enthusiasm as teachers to discover and develop new ideas and methods to enhance our effectiveness as engineering educators.

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