Experimental and Numerical Validation of DIN standard for polygonal shafts

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Abstract

With the advent of advanced manufacturing techniques such as multi-axis CNC machining, polygonal shafts are becoming a viable option for power transmission. Polygonal shafts are gaining popularity due to advantages such as self-centering, lack of stress concentration areas as in keys or splines, lower cost compared to splines, ability to press fit, and ease of assembly and disassembly. The polygonal shaft and hub are designed according to DIN 32711 (2009-03) and DIN 32712 (2009-03) for three and four lobe polygonal shaft respectively. These standards provide the formulae for the calculation of torsional shear stress and contact pressure for the shaft-hub connection. The purpose of this paper is to validate the results of the standard experimentally and numerically. To validate the result experimentally, a stacked rosette strain gages were used on three and four lobe shaft hub connections. The results were compared for maximum torsional shear stress and the errors were within 11 % for both profiles. The numerical results from a finite element model were found to be even more accurate with differences below 4 % for both profiles. These results validate the results from the DIN standard. However, the contact pressure values were found to be underreported by the DIN standard with the average contact stress being 6.21 and 5.07 times the estimated value by the DIN standard for three and four lobe shafts. The reason for the difference is the smaller area of contact in actual case than the DIN standard assumption, especially for the four-lobe profile due to expansion of the hub. The contact stress was found to cause local yielding in polygonal shafts and the chances of failure by dynamic loads need to be considered for design rather than designing solely with the current DIN standard.

1) Background

Polygonal shafts, as standardized by DIN, are industrial alternatives to keys and splines for power transmission. The industrial use of polygonal shafts for power transmission was limited in the past due to the need for dedicated machines to produce complex contours and lack of a standard design. The use of these shafts increased after the standardization of three lobe polygonal shaft, three lobe polygonal shaft, P3G by DIN 32711[1] and four lobe polygonal shaft, P4C, by DIN 32712[2] standard in 1979 and multi axis CNC machining. Polygonal shafts have advantages such as lower cost of manufacturing (40-50 % as compared to the spline joint), no stress risers as in keyways and splines, self-centering connections, and less vibration and noise as the connection can work in shrink fit unlike keyed shafts.[3] P4C shafts are used in sliding fit applications due to smaller normal axial stress and P3G shafts are used in press fit applications for larger torque transmission due to larger contact area which helps to distribute the contact stress.

Due to the complex conformal contact between the shaft and the hub, there is no analytical solution for the contact stress. The stress analyses of polygonal profiles were developed by various writers as Orlov[4] and Musyl[5] and were based on very strong approximations that did not accurately reflect the real stress and strain state. The aim of these analyses was to find the critical stress in hub since the hub was supposed to expand under torsion and fail. The procedure attempted to simplify the geometry of the polygon connection by analogous mechanical models. For example, Musyl used circular segments for profile approximation.[5] This approach of Musyl is usually referred by the polygonal connection manufacturers.[6] The current DIN standard follows the approach of Musyl with revision later on to include geometry for CNC capabilities and provides approximation of torsional shear stress and contact pressure and is used by industries to design the shafts for static torsional loads.[1,2] The DIN standard is divided into two parts. The first part deals with the Generalities and Geometry and the second part deals with the Calculation and Dimensioning.

2) Geometry

The polygonal profile is a special case of an epitrochoidal curve. The equation of the curve is given in equation (1), where, d_m is the mean diameter of the profile, e is the eccentricity, n is the periodicity (number of lobes), v is the parameter, $0 \leq v \leq 2\pi$.

Cartesian Coordinates:

$$\left.\begin{aligned} x(v) &= \left(\left(\frac{d_m}{2} - e\cos(nv)\right)\cos(v) - ne\sin(nv)\sin(v)\right) \\ y(v) &= \left(\left(\frac{d_m}{2} - e\cos(nv)\right)\sin(v) + ne\sin(nv)\cos(v)\right) \end{aligned}\right\} \tag{1}$$

Polar Coordinates:

$$\left.\begin{aligned} r(v) &= \sqrt{\left(\frac{d_m}{2} - e\cos(nv)\right)^2 + (ne\sin(nv))^2} \\ \phi(v) &= v + \tan^{-1}\left(\frac{ne\sin(nv)}{0.5d_m - e\cos(nv)}\right) \end{aligned}\right\} \tag{2}$$

The P3G profile is a harmonic curve as described by equation (1) and (2) while the P4C profile is the superposition of the four lobe profile as described by equation (1) or (2) and a circle with diameter of the grinding diameter as shown in Figure 1, where d_1 is the outer or grinding diameter, d_2 is the inner diameter and d_m is the mean diameter.

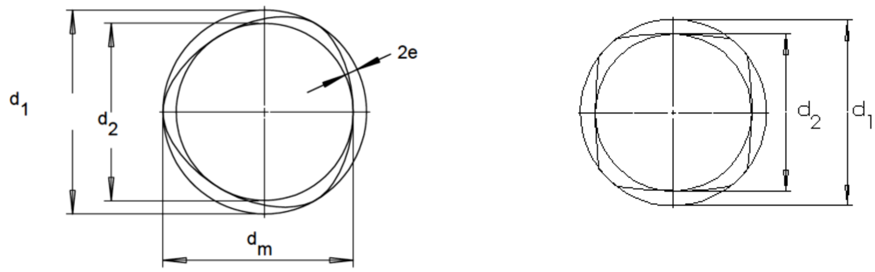

Figure 1: Profile parameters of the German DIN 32711 (P3G) and DIN 32712 (P4C) polygon standards.[1,2]

3) Load calculation

The load calculation for P3G and P4C shaft are described in DIN standard.[1,2] The three lobe, P3G shaft hub connection, is described by the nominal diameter d_m, eccentricity e and related eccentricity $\frac{e}{d_m}$. For P3G shaft hub connection subjected to a torsional load

$$T = \tau \times Z_p \tag{3}$$

is the applied torque, where, τ is the torsional shear stress and

$$Z_p = \frac{d_m+4e}{d_m+8e}\frac{A^4}{20I_p d_m} \tag{4}$$

is the polar moment of resistance of the cross-section where, d_m is the mean diameter and

$$A = \frac{\pi {d_m}^2}{4} - 4\pi e^2 \tag{5}$$

is the cross-sectional area of the profile.

$$I_p = \left(\frac{\pi {d_m}^4}{32} - \frac{3\pi {d_m}^2 e^2}{4} - 6\pi e^4\right) \tag{6}$$

is the polar moment of inertia of the cross-section.

$$P = \frac{T_{max}}{l(\pi e d_m + \frac{{d_m}^2}{20})} \tag{7}$$

is the surface pressure.

For P3G hub,

$$\left.\begin{aligned} t &= 1.44 \times \sqrt{\frac{T_{max}}{\sigma \times l}} \text{ for } d_m \leq 35 \text{ mm} \\ t &= 1.2 \times \sqrt{\frac{T_{max}}{\sigma \times l}} \text{ for } d_m > 35 \text{ mm} \end{aligned}\right] \tag{8}$$

is the distance between the end of the profile and the outer circumference of the hub where, *l* is the width of the hub and σ is the allowable tensile stress.

The four lobe, P4C shaft hub connection, is described by grinding diameter d_1, the inside diameter d_2 and ratio $\frac{d_1}{d_2}$ for defining the shaft and hub profiles. The nominal diameter of four lobe shaft is given by $d_m=d_2+2e$.For P4C shaft hub connection subjected to a torsional load,

$$T = \tau \times Z_p \tag{9}$$

is the applied torque where, τ is the torsional shear stress and

$$Z_p = 0.2{d_2}^3 \tag{10}$$

is the polar moment of resistance of the cross-section.

$$P = \frac{T_{max}}{l(\pi e_r d_r + \frac{{d_r}^2}{20})} \tag{11}$$

is the surface pressure where, $d_r = d_2+2e$ is the calculated theoretical diameter, e is the eccentricity, $e_r = \frac{d_1 - d_2}{4}$ is the theoretical eccentricity, , d_1 is the diameter of the circumscribed circle, *l* is the width of the hub, and σ is the allowable tensile stress.
For P4C Hub,

$$t = 0.7 \times \sqrt{\frac{T_{max}}{\sigma \times l}} \tag{12}$$

is the distance between the end of the profile and the outer circumference of the hub

Although, the power transmission shafts would mostly be subjected to a torsional bending load rather than a pure torsional load, the design considering only torsional load can be used for short shafts and comparatively lower bending loads.

4) Validation

Validation was conducted using experimental and numerical methods. Strain gages were used to conduct the experimental validation and the numerical validation was performed using finite element analysis. The validation process is explained as follows.

4.1) Experimental Validation

Strain gages were used to experimentally verify the results from DIN standard for P3G and P4C polygonal shaft and hub connections. For the purpose of the experiment, a polygonal shaft was subjected to a torsional load from the hub as shown schematically in Figure 2. The size of the shaft and hub are shown in Figure 3 for P3G and Figure 4 for P4C. The torsional shear stress developed in the shafts were evaluated using strain gages and the values were compared to the results from the DIN standard.

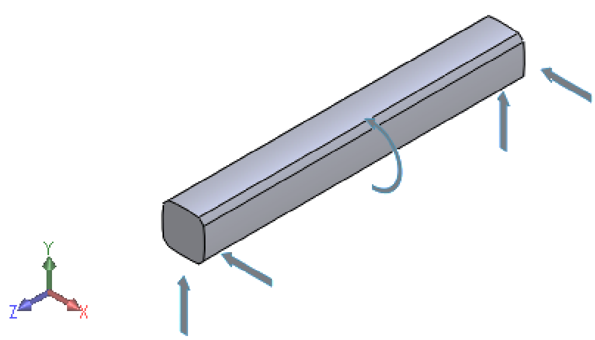

Figure 2: Loading for experimental setup

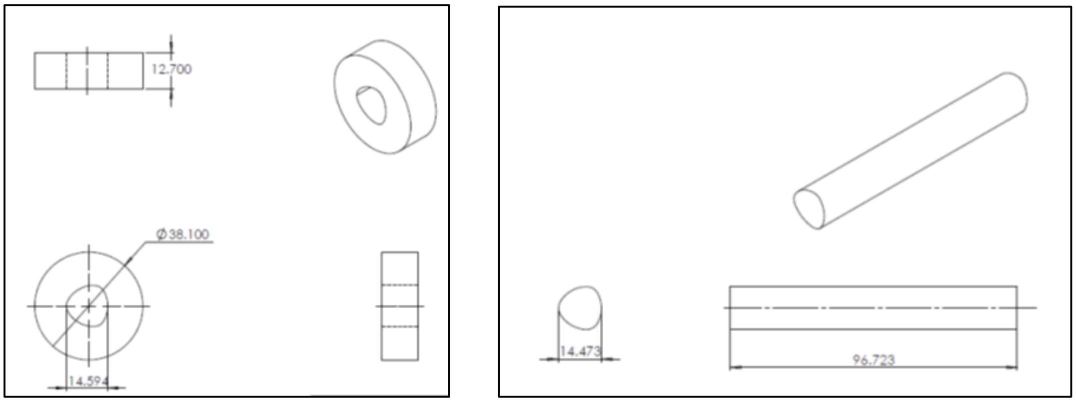

Figure 3: Dimension of P3G hub (left) and shaft (right)

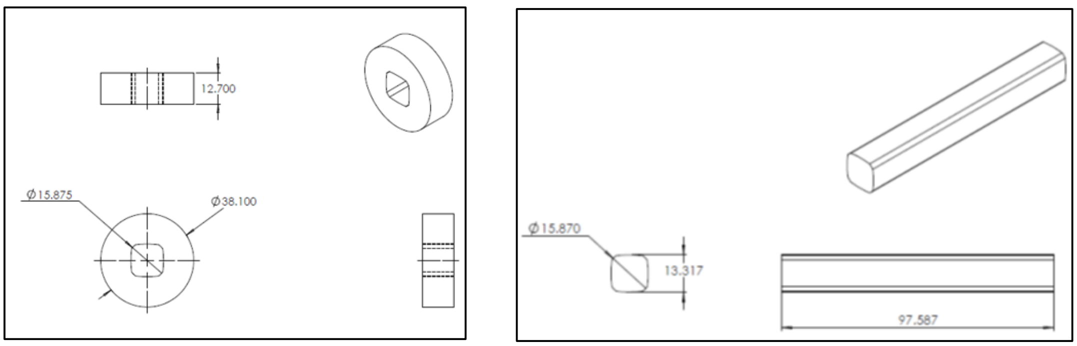

Figure 2: Dimension of P4C hub (left) and shaft (right)

The experiment consisted of positioning the strain gage on the shaft, performing the experiment, and analyzing the results. The steps are briefly explained as follows:

4.1.1) Position of strain gage

The objective of the experiment was to validate the stress calculated from DIN standard in the polygonal shaft hub connection. A stacked rectangular strain rosettes was used to measure strain for each polygonal shaft. The strain gage was aligned such that the middle strain gage was pointing in the axial direction, as shown schematically in Figure 5 (P3G) and Figure 6 (P4C), along the neutral axis of the shaft. The neutral axis was chosen to eliminate bending effects and to capture the maximum torsional shear stress, which occurs in the middle of the each side of the shaft.

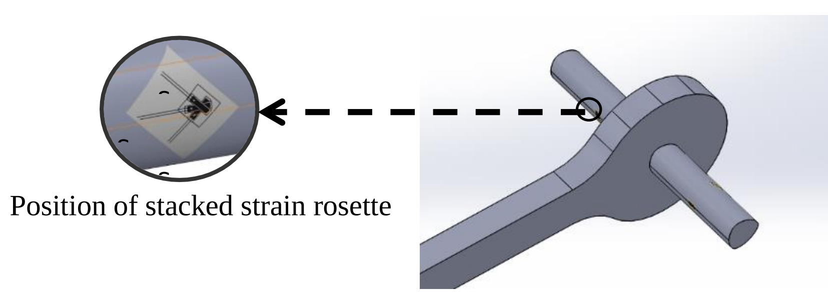

Figure 3: Position of strain gage in P3G shaft

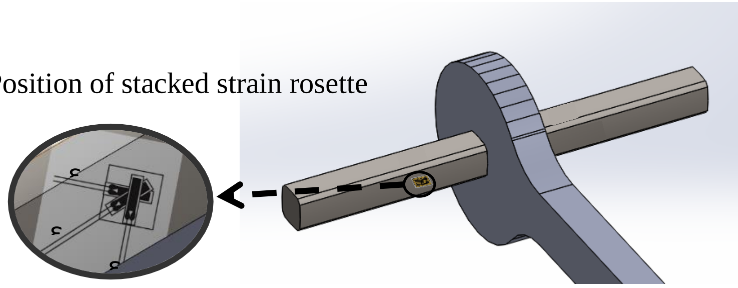

Figure 4: Position of strain gage in P4C shaft

4.1.2) Experimental Setup

For the experimental setup, the hub was placed in the middle of the shaft. Although, the shaft was 152.4 mm in length, only 101.6 mm was maintained between the two supports so as to avoid any slipping at the edge. The schematic of experimental setup is shown in Figure 7. A load of 6.82 kg (15 lb) was hung from the straight wrench with the help of chain of 0.794 kg so as to provide a torque of 22.13 Nm. Although, a small bending load is applied on the shaft as a result of force transformation, the position of the gage on the neutral axis of the bending load nullifies its effect on the specimen yielding a pure torsional load. The rosette strain gage and individual strain gage were composed of encapsulated 120 Ω constantan metal foil with gage length of 0.787 mm.

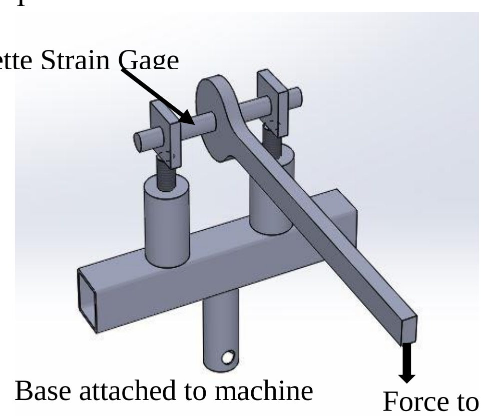

Figure 5: Schematic of experimental setup

4.1.3) Experimental Result

Table 1 shows the result form the experiment using 4140 steel with Young's Modulus of Elasticity (E) of 209 GPa, Poisson's ratio (v) of 0.3, and Modulus of rigidity (G) of 80 GPa. The strain gage ε_2 is the middle gage along the shaft axis. The reason for the axial normal strain is due to the slight rotation of the shaft around its axis due to the torsional load.

Table 1: Reading from the strain gage

Strain Gage	P3G Value (με)	P4C Value (με)
ε_1	163	-113
ε_2	17	52
ε_3	-148	127

The torsional shear stress calculated from strain values in Table 1 has been compared to the ones from the DIN standard in Table 2.

Table 2: Table showing the comparison of DIN and experimental results

Connection	Torsional shear stress (MPa)		
	DIN	**Strain Gage**	**% Error**
P3G	22.22	24.93	10.87
P4C	23.33	21.50	8.51

From Table 2, the result from the strain gage and the DIN standard are found to be close to each other with percentage error below 11 %. The reason for the discrepancy is due to the small rotation of the shaft due to the load. Since, the results are below 11 % and do not vary more than 2.71 MPa from each other, the DIN standard is fairly accurate in representing the torsional shear stress in the polygonal shaft.

4.2) Numerical Validation

The conformal contact between the shaft and the hub in a polygonal connection leads to a complex tri-axial stress, for which there is no analytical solution. The values provided from the DIN standard are the maximum torsional shear stress and the contact pressure for a pure torsional load. To verify the values from the DIN standard, a nonlinear finite element model of P3G and P4C connections were analyzed using ANSYS 15.0.7 software. To match the requirements of the DIN standard, a line to line fit between the shaft and the hub without considering the friction were modeled. The finite element model was made with a hub and shaft of size as shown in Figure 3 and 4 for P3G and P4C with the hub at the center of the shaft and fixed in all directions at the edge except axial. The P3G profile had a nominal diameter of 14.478 mm and eccentricity of 0.508 mm and the P4C profile had outside diameter of 15.875 mm, inside diameter of 13.335 mm and eccentricity of 1.905 mm. The considerations for the model were as follows:

- Only half of the model along the axis was considered for analysis as shown in Figure 8 and symmetry was applied for the other half.
- The contact solution formulation chosen was augmented Lagrangian that takes into consideration advantages of both the penalty and Lagrangian method
- The contact behavior was chosen as symmetrical, which meant that neither the target nor the contact can penetrate each other
- The number of iterations was set to 100 with the convergence criteria of 0.5 %
- Frictionless contact was considered.
- The body was meshed with a lower order hexahedron mesh (Solid 185) with contact elements of 0.27 mm. The mesh are shown in Figure 8.

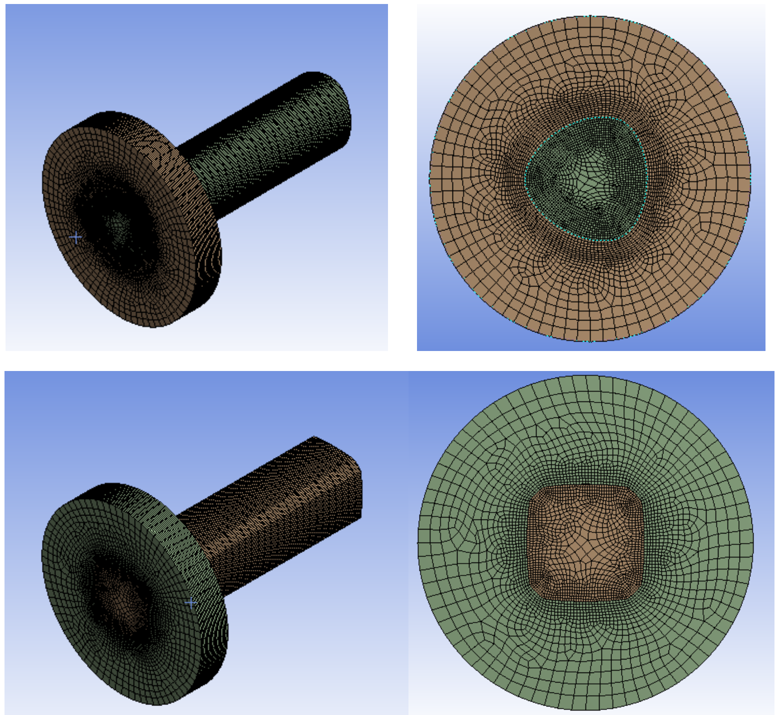

Figure 6: Meshing of three lobe and four lobe connections

A torsional load of 80 Nm was applied around the circumference of the hub while fixing the shaft ends in all directions except axial. The result from the FEA are compared to the result from the DIN standard and are shown in Table 3.

Table 3: Comparison of FEA and DIN standard for torsional loading of 80 Nm

Profile	Maximum torsional shear stress (MPa)			Contact Stress (MPa)			
	FEA	DIN	Percentage Difference (%)	FEA			DIN
				Maximum	Average excluding zero stress values	Average including zero stress values	
P3G	154.95	160.50	3.52	2111.30	1147.72	407.04	226.51
P4C	165.54	168.69	1.88	2938.00	800.53	142.32	128.82

From the comparison Table 2, the result of FEA seems to be very close to the theoretical maximum torsional shear stress, with percentage difference being only 3.52 % and 1.88 % for P3G and P4C shaft respectively. This shows that the result from the DIN standard can be trusted to find the torsional shear stress developed in the shaft.

Polygonal shafts rarely fail from static loads. One of the common failure is by fretting fatigue.[8] The shaft and hub connection edge act as a site of crack initiation and fracture and pitting for the polygonal shafts. The contact stress developed at the shaft hub interface is the reason for the crack initiation. Contact stress is the major stress in the shaft hub connection and is highest at the edge of the shaft and hub connection.[8] Hence, validation of the contact stress values from the DIN standard are important for design.

For the current analysis, the contact stress at the edge of shaft hub interface, as in the DIN standard, has been compared to the FEA model. The interface has certain regions where the contact occurs. Although a line to line fit was modeled, the expansion of the hub as a result of the torque causes expansion of the hub and there are distinct areas of contact. The number of area of contact is equal to the number of lobes in the profile. The contact stress around the edge of the hub contact are shown in Figure 9.

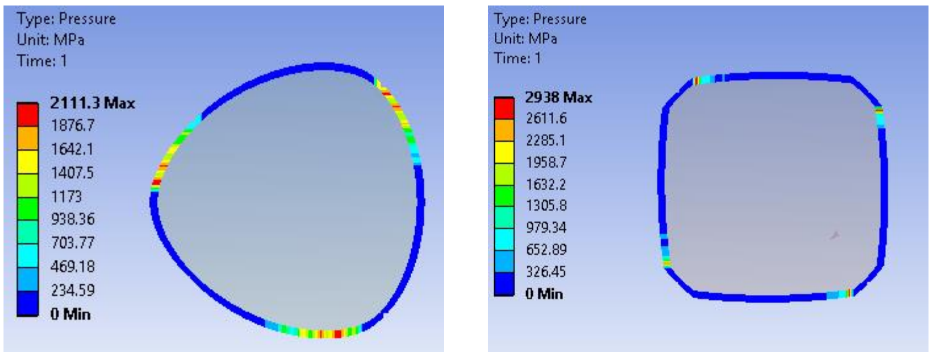

Figure 7: Contact stress distribution along the edge of the shaft hub connection

For the P4C shaft, FEA shows the maximum contact stress to be much larger than the contact stress from the DIN standard (22.81 times the contact stress value from the DIN standard). The DIN standard calculates the average contact stress and is much smaller than the maximum stress. By taking the average of the contact stress values excluding zero (assuming no contact in positions where stress is zero), the average is still found to be much higher than the DIN standard (6.21 times the contact stress value from the DIN standard). But, if the average of all the contact stress values along the edge of the shaft hub connection are considered, the values are only 1.1 times the values estimated by the DIN standard, with a difference of only 9.96 %.

For P3G shaft, the results are similar to the P4C shaft. The maximum contact stress is 9.32 times that of the DIN standard. The average contact stress excluding zero stress values (i.e. without contact) is 5.07 times the DIN standard and the average contact stress including the whole closed edge is 1.8 times the DIN standard, with a difference of 57 %.

The deviation of the average contact stress from the DIN standard is due to the theoretical area of contact used in DIN standard. In an actual loading, the hub expands as a result of the torsional load, even in the transition fit and there are distinct area of contact rather than the whole area as seen from Figure 9. So, the distinct smaller area will have to bear greater contact pressure leading to the discrepancy. The maximum contact stress in P4C shaft is 1.39 times that of the P3G shaft from FEA. However, the average contact stress is smaller in the P4C shaft agreeing with the result from the DIN standard. The reason for greater maximum contact stress in P4C shaft is due to the larger maximum pressure angle than the P3G shaft that led to smaller contact area. A careful look at the cross section of the P4C shaft in Figure 1 shows the sharper edge at the discontinuity of the epitrochoidal curve and the grinding circle. The sharper edge has the largest pressure angle and the shaft hub connection has smaller contact area and larger contact pressure as seen from Figure 9 and 10.

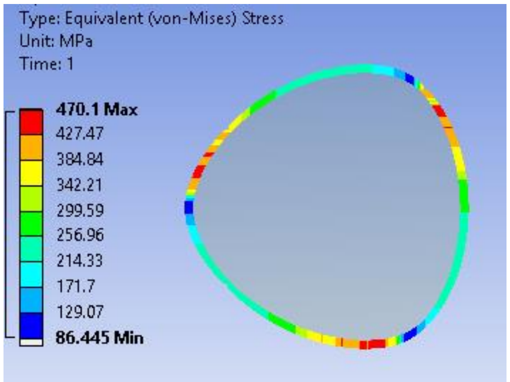

Figure 8: Distribution of von Mises stress along the edge

The von Mises stress distribution for the P3G and P4C shaft hub edge are shown in Figure 10 and follow the same distribution as the contact stress inferring that the contact stress is the major contributor for von Mises stress. It is interesting to note that the von Mises stress is higher than the yield stress for P4C shafts, but the structure does not fail. The reason for this is the localized nature of the contact stress that causes local yielding but does not cause fracture. The polygonal shafts may ultimately develop cracks and pits as a result of these local yielding and fail by fretting fatigue[8] and pitting corrosion.

Based on the result, the DIN standard is found unable to capture the actual contact stress that develops at the shaft hub interface. The values from the DIN standard are lower than the FEA and designing a shaft based on the DIN standard may cause the structure to fail by aforementioned fatigue and pitting much faster than anticipated in a normal circular shaft. Another reason for the not relying on the DIN standard for the contact stress is the lack of results for different fits and consideration of friction, which has also been depicted by Winterfeld[9].

5) Conclusion

The DIN standard was accurate in predicting the maximum torsional stress developed in polygonal shaft with error of 10.87 and 8.51 % for P3G and P4C shafts respectively from experimental validation using strain gage. The maximum torsional shear stress was also validated from Finite Element Analysis with difference of 3.52 % and 1.88 % for P3G and P4C profiles respectively.

For the contact pressure, the results were much different with the results from FEA being higher than the one from the DIN standard. The maximum contact stress was 22.81 times and 9.32 times the DIN standard for P3G and P4C shafts respectively and the average stress was 6.21 and 5.07 times the values predicted by the DIN standard. This shows that the DIN standard cannot be relied for determining the contact stress. However, since the contact stresses are highly local and don't cause the structure to fail, the possibility of failure by mechanisms as fretting fatigue and pitting is high. Therefore, and the structure needs to be designed using higher safety factor if DIN standard is to be relied upon. The standard should state the possibility of failure from dynamic load because of the higher contact stress even though the torsional shear stress is low.

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